

# PROCESS DYNAMICS AND CONTROL

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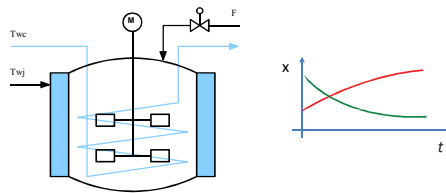


## RECAP

- ◉ Importance of dynamic modeling
- ◉ Importance of process control
- ◉ Examples of control systems
- ◉ Basic definitions
  - Controlled variable, manipulated variable, disturbance variable, set point.

## APPLICATIONS OF DYNAMIC MODELING

- Improve process understanding
- Train operating personnel
- Develop control strategies
- Optimize operating conditions

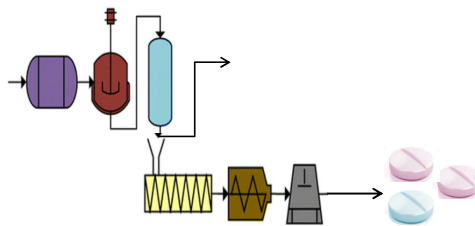


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## OPTIMIZING OPERATING CONDITIONS

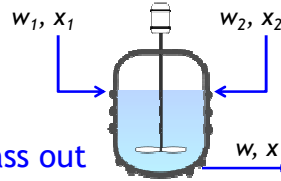
- Examples:
  - Minimize startup time
  - Minimize startup waste
  - Maximize yield in a batch process
- We need a dynamic model for optimization



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## BLENDING TANK MODEL



- Steady-state model

- Assumptions?

$$0 = \text{Rate of mass in} - \text{Rate of mass out}$$

$$w_1 + w_2 - w = 0$$

$$w_1x_1 + w_2x_2 - wx = 0$$

- Dynamic model

$$\text{Rate of accumulation} =$$

- Assumptions?

$$\text{Rate of mass in} - \text{Rate of mass out}$$

$$\frac{d(\rho V)}{dt} = w_1 + w_2 - w$$

$$\frac{d(\rho Vx)}{dt} = w_1x_1 + w_2x_2 - wx$$

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## ORDINARY-DIFFERENTIAL EQUATIONS

- Dynamic models yield differential equations

- Time as an independent variable

- This course is limited to ordinary differential equations (ODEs)

- $p$ : constant parameters

- $x$ : state variables

$$\frac{dx}{dt}(t) = f(x(t), p, t)$$

- Find **state variables** and **parameters** in the blending example.

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## CAN WE SIMPLIFY THE MODEL?

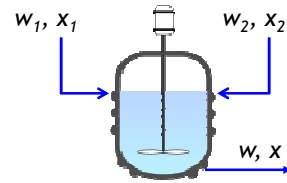
- More assumptions?

- Constant density

$$\rho \frac{d(V)}{dt} = w_1 + w_2 - w$$

$$\rho \frac{d(Vx)}{dt} = w_1 x_1 + w_2 x_2 - wx$$

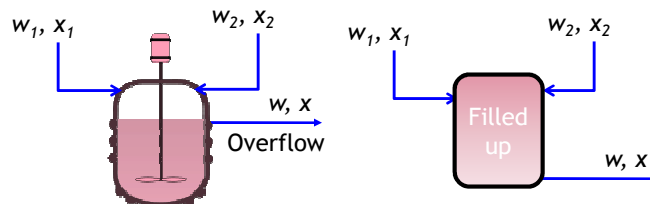
$$\rho \left( V \frac{d(x)}{dt} + x \frac{d(V)}{dt} \right) = w_1 x_1 + w_2 x_2 - wx$$



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## SIMPLIFY THE MODEL EVEN FURTHER

- Assume constant volume
- How?
  - Overflow line
  - Closed tank that is filled to capacity
  - Nearly perfect control of level



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## HOW TO SOLVE THE DYNAMIC MODEL

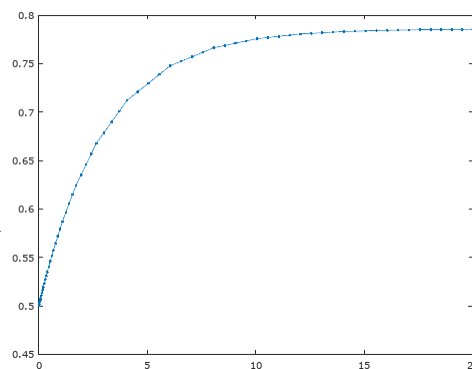
- ⦿ Analytically
  - If model is simple enough
  
- ⦿ Numerically
  - Used in most practical cases

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## DYNAMIC RESPONSE OF SYSTEM

- ⦿ Sudden change in inlet composition  $x$  from 0.4 to 0.8.
- ⦿ Initial condition
- ⦿ Dynamic solution
  - Transient behavior
  - Final steady state

See Example 2.1 of Seborg's book



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## DEGREES OF FREEDOM (DOF) ANALYSIS

- How to know if the model can be solved?

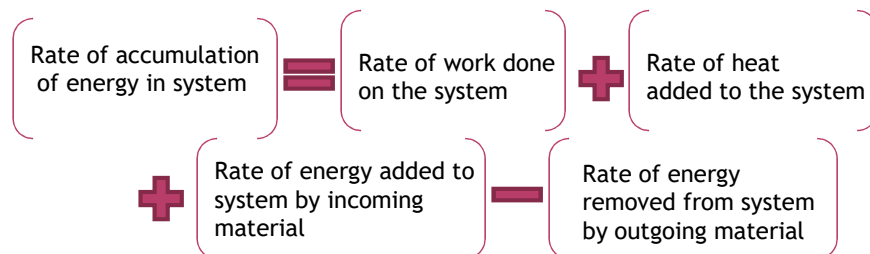
Linear system of equations  $y = Ax$

For General systems of equations  $N_F = N_V - N_E$

- Exactly Specified
- Underspecified
- Overspecified

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## DYNAMIC ENERGY BALANCE



Total energy in the system

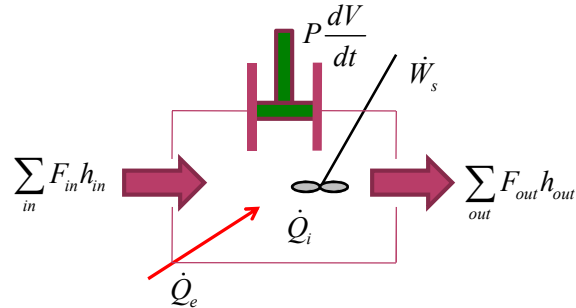
$$E_{tot} = U + E_K + E_P$$

✓ Changes in **kinetic** and **potential** energies can usually be ignored in process modeling

**Exceptions?**

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## ENERGY BALANCE DERIVATION



$$\frac{dU}{dt} = \dot{Q} + \dot{W} + \sum_{in} F_{in} h_{in} - \sum_{out} F_{out} h_{out}$$

$$\dot{W} = \dot{W}_{shaft} - P \frac{dV}{dt}$$

$$\dot{Q} = \dot{Q}_e + \dot{Q}_i$$

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## ENERGY BALANCE DERIVATION

$$\frac{dU}{dt} = \dot{Q} + \dot{W}_s - P \frac{dV}{dt} + \sum_{in} F_{in} h_{in} - \sum_{out} F_{out} h_{out}$$

► Constant-volume (rigid vessel)  $\frac{dU}{dt} = \dot{Q} + \dot{W}_s + \sum_{in} F_{in} h_{in} - \sum_{out} F_{out} h_{out}$

$$\frac{dU}{dt} = \frac{dH}{dt} - \frac{d(PV)}{dt} = \dot{Q} + \dot{W}_s - P \frac{dV}{dt} + \sum_{in} F_{in} h_{in} - \sum_{out} F_{out} h_{out}$$

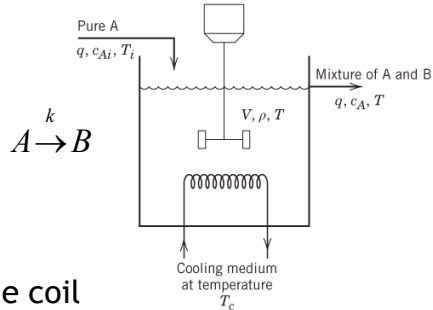
► Constant-pressure

$$\frac{dH}{dt} - P \frac{dV}{dt} - V \frac{dP}{dt} = \dot{Q} + \dot{W}_s - P \frac{dV}{dt} + \sum_{in} F_{in} h_{in} - \sum_{out} F_{out} h_{out}$$

➔  $\frac{dH}{dt} = \dot{Q} + \dot{W}_s + \sum_{in} F_{in} h_{in} - \sum_{out} F_{out} h_{out}$

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## EXAMPLE: DYNAMIC NON-ISOTHERMAL CSTR



### Some assumptions

- Constant volume
- Constant density
- $T_c$  constant through the coil

Figure 2.6 A nonisothermal continuous stirred-tank reactor.

$$V\rho C \frac{dT}{dt} = wC(T_i - T) + (-\Delta H_R)Vk c_A + UA(T_c - T)$$

$$k = k_0 e^{\left(\frac{-E}{RT}\right)}$$

Nonlinear ODE? Why?

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## EXAMPLE: DYNAMIC NON-ISOTHERMAL CSTR

Change in cooling water temperature

- I. -10 K
- II. +5 K

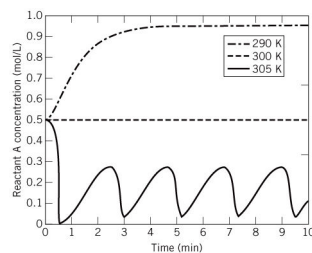


Figure 2.8 Reactant A concentration variation with step changes in cooling water temperature to 305 and 290 K.

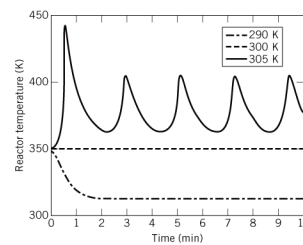


Figure 2.7 Reactor temperature variation with step changes in cooling water temperature from 300 to 305 K and from 300 to 290 K.

Response **changes dramatically** with **direction & magnitude** of change (typical for nonlinear dynamics)

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**ODE SIMULATION USING OCTAVE  
(MATLAB!)**

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