PROCESS DYNAMICS AND CONTROL

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RECAP

- Dynamic energy balance
- How to analyze dynamic systems?

LAPLACE TRANSFORMS

- A mathematical tool for convenient analysis of dynamic systems.

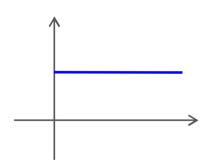
- Definition

$$F(s) = \mathscr{L}[f(t)] = \int_0^{\infty} f(t)e^{-st}dt$$

LAPLACE TRANSFORMS OF COMMON FUNCTIONS

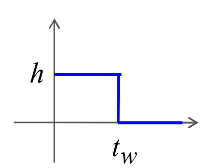
Step function

$$S(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 & \text{for } t \geq 0 \end{cases} \qquad \mathfrak{L}\left[S(t)\right] = \frac{1}{s}$$



Rectangle pulse function

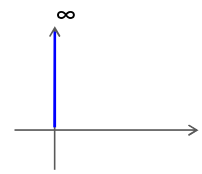
$$f(t) = \begin{cases} 0 & \text{for } \; t < 0 \\ h & \text{for } \; 0 \leq t < t_w \\ 0 & \text{for } \; t \geq t_w \end{cases} \qquad F(s) = \frac{h}{s}\left(1 - e^{-t_w s}\right)$$

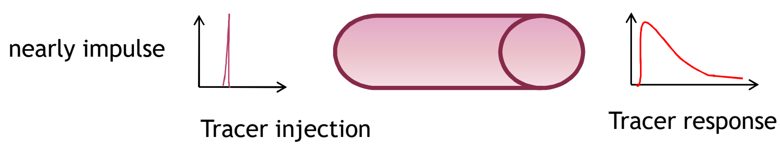


LAPLACE TRANSFORMS OF COMMON FUNCTIONS

Impulse function (Dirac Delta)

$$\delta(t) = \lim_{t_w \to 0} f(t) = \begin{cases} 0 & \text{for } t < 0 \\ \frac{1}{t_w} & \text{for } 0 \le t < t_w \\ 0 & \text{for } t \ge t_w \end{cases} \qquad F(s) = 1$$





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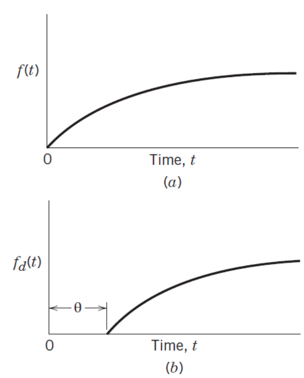
LAPLACE TRANSFORMS OF COMMON FUNCTIONS

Time delay

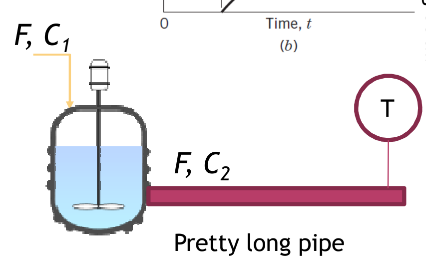
$$f_d(t) = f(t-\theta)S(t-\theta), \ \theta = \text{time delay}$$

$$\mathfrak{L}\left[f_d(t)\right] = e^{-\theta s} F(s)$$

- Transportation delay between the tank and the sensor
- Measurement delay (take samples and analyze them in the lab)



Process Dynamics & Control. Seborg et al.



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LAPLACE TRANSFORM OF DERIVATIVE

$$\mathfrak{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f^{(1)}(0) - \ldots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

If all initial conditions are zero $f(0) = f^{(1)}(0) = \ldots = f^{(n-1)}(0).$

$$\mathfrak{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s)$$

Application: nominal steady-state with deviation variables (to be discussed later)

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LAPLACE TRANSFORM OF INTEGRAL

$$\mathfrak{L}\left[\int_0^t f(\tau) d\tau\right] = \frac{F(s)}{s}$$

Prove using integration by part…

Application: integrating systems; PID controllers

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PROPERTIES OF LAPLACE TRANSFORM

- Laplace and its inverse are linear operators.

$$\mathfrak{L}\left[ax(t)+by(t)\right]=a\mathfrak{L}\left[x(t)\right]+b\mathfrak{L}\left[y(t)\right]$$
$$=aX(s)+bY(s)$$

$$\mathfrak{L}^{-1}\left[aX(s)+bY(s)\right]=ax(t)+by(t)$$

SOLVING ODES USING LAPLACE TRANSFORM

1) Take Laplace of both sides
2) Rearrange
3) Take inverse Laplace

I) $$\frac{dx}{dt}=-2x,\quad x(0)=3$$

II) $$\frac{dx}{dt}=-2x+1,\quad x(0)=3$$

PARTIAL FRACTION EXPANSIONS

- Expand the complex Laplace function into simpler terms that are given in the table.

$$Y(s) = \frac{s+5}{(s+1)(s+4)}$$

Perform a partial fraction expansion (PFE)

$$\frac{s+5}{(s+1)(s+4)} = \frac{\alpha_1}{s+1} + \frac{\alpha_2}{s+4}$$

$$\alpha_1, \quad \alpha_2 \,?$$

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PARTIAL FRACTION EXPANSION

- Real, distinct roots

$$Y(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{\pi_{i=1}^{n}(s+b_i)}$$

$D(s)$: an n-th order polynomial with all real, distinct roots $(s = -b_i)$

$$Y(s) = \frac{N(s)}{\pi_{i=1}^{n}(s+b_i)} = \sum_{i=1}^{n} \frac{\alpha_i}{s+b_i}$$

- Repeated roots

$$Y(s) = \frac{\text{N(s)}}{(s+b)^r} = \frac{\alpha_1}{(s+b)} + \frac{\alpha_2}{(s+b)^2} + \ldots + \frac{\alpha_r}{(s+b)^r}$$

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PARTIAL FRACTION EXPANSION

- Complex roots $$Y(s) = \frac{c_1 s + c_0}{s^2 + d_1 s + d_0}, \quad \frac{d_1^2}{4} < d_0$$
- Completing the square $$Y(s) = \frac{c_1 s + c_0}{s^2 + d_1 s + d_0} = \frac{a_1(s+b) + a_2}{(s+b)^2 + \omega^2}$$
- Example

$$Y(s) = \frac{s+2}{s^2+s+1} = \frac{s+2}{s^2+s+0.25-0.25+1}$$
$$= \frac{s+2}{(s+0.5)^2+0.75} = \frac{(s+0.5)+1.5}{(s+0.5)^2+(\frac{\sqrt{3}}{2})^2}$$

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CLASS EXERCISE

- Solve using Laplace transform

$$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 1, \quad y(0) = \dot{y}(0) = \ddot{y}(0) = 0$$

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TWO USEFUL THEOREMS

- Final-value theorem
  - Useful for getting steady-state values without having to solve the ODE

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0}\left[sY(s)\right]$$

If the limit exists for all Re(s)≥0

- Initial-value theorem

$$\lim_{t\to 0} y(t) = \lim_{s\to \infty}\left[sY(s)\right]$$