PROCESS DYNAMICS AND CONTROL

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CHAPTER 5: RESPONSE OF MORE COMPLICATED PROCESSES

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POLES OF TRANSFER FUNCTION

- Example (assume underdamped system)

$$G(s) = \frac{K}{s(\tau_1 s + 1)(\tau_2^2 s^2 + 2\zeta\tau_2 s + 1)} \qquad (6\text{-}1)$$

- **Response Modes**

  Applying partial fraction expansion
  - Constant term
  - Exponential term
  - Sine/cosine terms
  - Additional terms (from input function)

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COMPLEX S PLANE

- Indicate roots of characteristic polynomial

$$s_1 = 0$$

$$s_2 = -\frac{1}{\tau_1}$$

$$s_3 = -\frac{\zeta}{\tau_2} + j\frac{\sqrt{1-\zeta^2}}{\tau_2} \qquad (6\text{-}2)$$

$$s_4 = -\frac{\zeta}{\tau_2} - j\frac{\sqrt{1-\zeta^2}}{\tau_2}$$

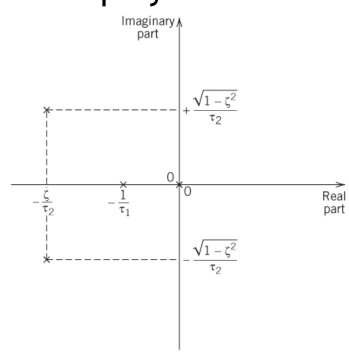

Figure 6.1 Poles of $G(s)$ (Eq. 6-1) plotted in the complex s plane (× denotes a pole location).

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MORE COMPLEX PROCESSES: EXAMPLE

- Lead-lag element

$$\tau_1 \frac{dy}{dt} + y = K\left(\tau_a \frac{du}{dt} + u\right) \qquad (6\text{-}3)$$

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RESPONSE OF MORE COMPLICATED PROCESSES

- General form of transfer function

$$G(s) = \frac{\sum_{i=0}^{m} b_i s^i}{\sum_{i=0}^{n} a_i s^i}$$

$$G(s) = \frac{b_m (s - z_1)(s - z_2)\ldots(s - z_m)}{a_n (s - p_1)(s - p_2)\ldots(s - p_n)}$$

- Recall condition for physical realizability?

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- Gain/time constant form

$$G(s) = K\frac{(\tau_a s + 1)(\tau_b s + 1)\cdots}{(\tau_1 s + 1)(\tau_2 s + 1)\cdots} \qquad (6\text{-}8)$$

- What does this form tell us?

- Any effect of zeros on the poles?

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LEAD-LAG ELEMENT REVISITED

$$\tau_1 \frac{dy}{dt} + y = K\left(\tau_a \frac{du}{dt} + u\right) \qquad (6\text{-}3)$$

$$G(s) = \frac{K(\tau_a s + 1)}{\tau_1 s + 1} \qquad (6\text{-}4)$$

- Step input

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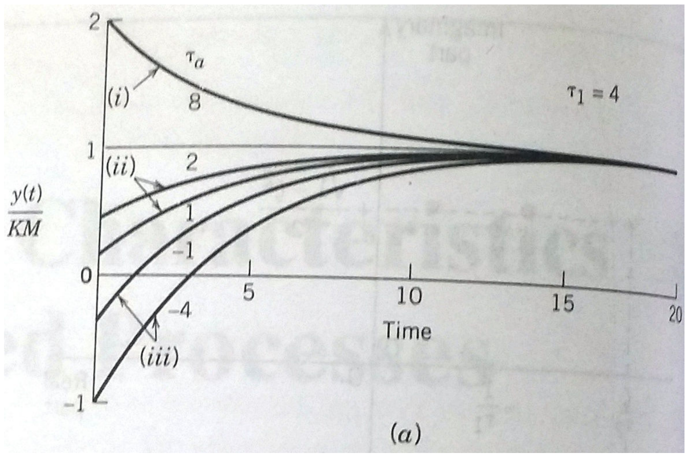
RESPONSE TRAJECTORIES



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2[ND]-ORDER WITH NUMERATOR DYNAMICS

- Transfer function

$$G(s) = \frac{K(\tau_a s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} \qquad (6\text{-}11)$$

- Time response

$$y(t) = KM\left(1 + \frac{\tau_a - \tau_1}{\tau_1 - \tau_2} e^{-t/\tau_1} + \frac{\tau_a - \tau_2}{\tau_2 - \tau_1} e^{-t/\tau_2}\right)$$

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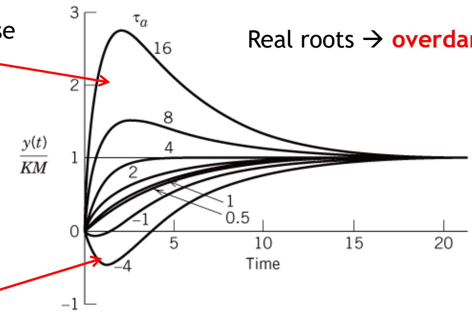
2[ND]-ORDER WITH NUMERATOR DYNAMICS

Overshoot response

Real roots → **overdamped** response



Inverse response

Figure 6.2 Step response of an overdamped second-order system (Eq. 6-11) for different values of τ_a ($\tau_1 = 4, \tau_2 = 1$).

Case i:	$\tau_a > \tau_1$	$(\tau_a = 8,\ 16)$
Case ii:	$0 < \tau_a \le \tau_1$	$(\tau_a = 0.5,\ 1,\ 2,\ 4)$
Case iii:	$\tau_a < 0$	$(\tau_a = -1, -4)$

INVERSE RESPONSE: DAY-TO-DAY EXAMPLE