

PROCESS DYNAMICS AND CONTROL

Ali M. Sahlodin
Department of Chemical Engineering
Amirkabir University of Technology
Fall 1397 S.H



RECAP

- ◉ Root locus method

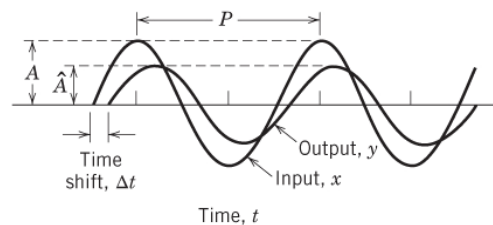
STABILITY METHODS SO FAR

- Root locus:
 - Polynomials only
 - What if we have delay in the transfer function?

3

FREQUENCY RESPONSE ANALYSIS

- Long time first-order sine response (blending tank)
 - After transient response (exponential terms) has disappeared



4

AMPLITUDE RATIO AND PHASE ANGLE: SHORTCUT METHOD

- ⦿ Amplitude

$$s = j\omega \quad G(j\omega) = K_1 + K_2j$$

$$|G| = AR = \sqrt{K_1^2 + K_2^2}$$
- ⦿ Phase angle

$$\phi = \angle G = \arctan \frac{K_2}{K_1}$$

5

EXAMPLE: OBTAIN PHASE ANGLE AND AMPLITUDE RATIO

$$G(s) = \frac{1}{\tau s + 1} \quad \rightarrow \quad G(j\omega) = \frac{1}{\tau j\omega + 1} = \frac{1}{j\omega\tau + 1} \quad (14-10)$$

$$G(j\omega) = \frac{-j\omega\tau + 1}{(j\omega\tau + 1)(-j\omega\tau + 1)} = \frac{-j\omega\tau + 1}{\omega^2\tau^2 + 1}$$

$$= \frac{1}{\omega^2\tau^2 + 1} + j \frac{(-\omega\tau)}{\omega^2\tau^2 + 1} = R + jI \quad (14-11)$$

$$AR = \sqrt{\frac{(1 + \omega^2\tau^2)}{(\omega^2\tau^2 + 1)^2}} = \frac{1}{\sqrt{\omega^2\tau^2 + 1}} \quad (14-13a)$$



$$\phi = \angle G(j\omega) = \tan^{-1}(-\omega\tau) = -\tan^{-1}(\omega\tau) \quad (14-13b)$$

6

AR AND PHASE ANGLE FOR COMPLEX TF

$$G(s) = \frac{G_a(s)G_b(s)G_c(s)\cdots}{G_1(s)G_2(s)G_3(s)\cdots} \quad (14-15)$$

$$G(j\omega) = \frac{G_a(j\omega)G_b(j\omega)G_c(j\omega)\cdots}{G_1(j\omega)G_2(j\omega)G_3(j\omega)\cdots} \quad (14-16)$$



$$|G(j\omega)| = \frac{|G_a(j\omega)||G_b(j\omega)||G_c(j\omega)|\cdots}{|G_1(j\omega)||G_2(j\omega)||G_3(j\omega)|\cdots} \quad (14-17a)$$

$$\begin{aligned} \angle G(j\omega) &= \angle G_a(j\omega) + \angle G_b(j\omega) + \angle G_c(j\omega) + \cdots \\ &- [\angle G_1(j\omega) + \angle G_2(j\omega) + \angle G_3(j\omega) + \cdots] \end{aligned} \quad (14-17b)$$

7

BODE DIAGRAMS

- Plot AR and phase angle versus frequency

$$G(s) = \frac{1}{\tau s + 1}$$

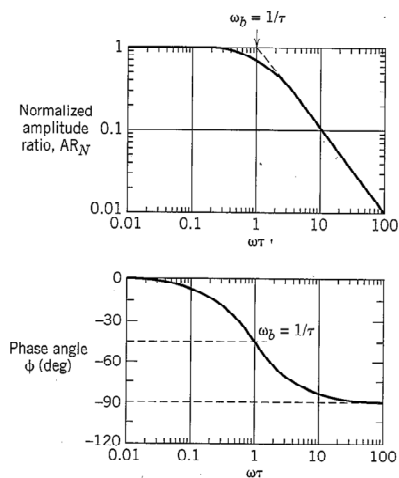


Figure 14.2 Bode diagram for a first-order process

8

BODE DIAGRAM: TIME DELAY

- No impact on AR
- Increasing phase lag
 - Impact on stability

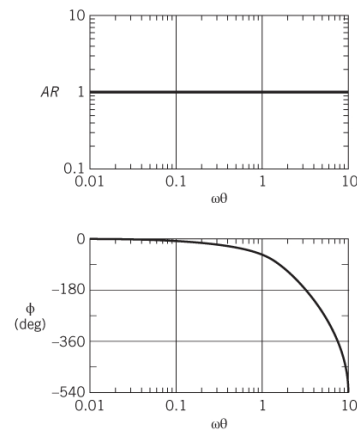


Figure 14.4 Bode diagram for a time delay, $e^{-t\tau}$.